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# DYNAMICS AND OPTIMAL CONTROL OF FLEXIBLE HEAT-EXCHANGER NETWORKS

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**Abstract** - A distributed-parameter model of multi-tube, single-pass heat exchangers are used to construct the dynamic model of a heat-exchanger network (HEN). Based on the control-vector parameterization technique, open-loop (time-dependent bypass manipulation) and closed-loop (target-temperature dependent bypass manipulation) centralized optimal-control schemes are developed and tested. The parameterizations are done in such a way that the manipulated variables approach and eventually match the values obtained from static optimization which refers to an algebraic model of the HEN. Manipulation of the bypasses linearly in time up to their optimal values results in a quite satisfactory dynamic response of the HEN and eliminates temporary violations of the target-temperature control-precision constraints.

## INTRODUCTION

Most steady-state synthesis methods (e.g., the pinch method) generate HENs for nominal values of stream supply temperatures and flow rates. However, a HEN, synthesized for the nominal conditions, must be flexible/resilient to desired/undesired changes in supply temperatures and flow rates. Temperatures and/or flow rates of incoming streams may introduce disturbances to a HEN owing to uncontrolled upsets in upstream process units and affect target streams, and hence, the operations in downstream process units. Thus, temperatures of target streams of a HEN should be tightly controlled for the safety of downstream operations. A synthesized HEN must be flexible and resilient to disturbances in its source streams to be able to control the network`s target-stream temperatures at desired values or within a given permissible range. The degrees of freedom required to achieve control objectives can be introduced by bypass streams and/or split streams.

Various authors investigated flexibility of HENs (Colberg and Morari, 1988), however, few authors studied flexibility and controllability of HENs simultaneously. Mathisen et al. (1991) reviewed different controllability measures and showed how to select bypasses and their appropriate pairings with targets. Mathisen (1994) suggested several optimization problems and proposed to take controllability into account by adding control-related constraints in formulating the flexibility problem. Papalexandri and Pistikopoulos (1994a, 1994b) presented a systematic framework for the synthesis and retrofit of flexible and structurally controllable HENs, and developed total disturbance-rejection criterion which was included within a MINLP model, based on a multiperiod hyperstructure network representation. Konukman et al. (1995) demonstrated the possibility of creating alternate disturbance-propagation paths via bypass-fraction manipulations by imposing different range constraints on hot and cold targets.

In this work; a dynamic model of a HEN is constructed using rigorous distributed-parameter models of its heat exchangers and two centralized optimal-control schemes (open-loop: time-dependent bypass manipulation and closed-loop: target-temperature dependent bypass manipulation) are developed and tested.

## DYNAMIC MODEL OF HEAT EXCHANGERS

Single-tube-pass, shell-and-tube heat exchangers were chosen as the exchanger type. The design parameters were selected such that the resulting exchangers were close to off-the-shelf-type heat exchangers. For this purpose, a detailed design algorithm was developed (Raman, 1985; Boyacı, 1995). Energy balance over differential volume element covering the tube and shell fluids, and the tube and shell walls was applied in obtaining the distributed-parameter model of the heat exchangers (Friedly, 1972), given by Eqs.1-4, to construct the dynamic model of the HEN (Boyacı, 1995).

Utilizing the "method of lines" numerical technique, these PDEs were discretized in z-direction using "five-point biased-upwind" finite-difference formulas, which eliminated steady-state offset problem, and thus, converted into ODEs, which were then integrated using the software package LSODES. To obtain the HEN model, the PDEs of the individual exchangers were interconnected through boundary conditions.

$$\frac{\partial T_H}{\partial t} = -v_H \frac{\partial T_H}{\partial z} + \frac{h_H a_H}{A_H \rho_H C_{P_H}}\left(T_T - T_H\right) \tag{1}$$

$$\frac{\partial T_C}{\partial t} = v_C \frac{\partial T_C}{\partial z} + \frac{h_C \sum_{N_T} a_C}{A_C \rho_C C_{P_C}}\left(T_T - T_C\right) + \frac{h_S a_S}{A_C \rho_C C_{P_C}}\left(T_S - T_C\right) \tag{2}$$

$$\frac{\partial T_T}{\partial t} = \frac{h_H a_H}{A_T \rho_T C_{P_T}}\left(T_H - T_T\right) + \frac{h_C a_C}{A_T \rho_T C_{P_T}}\left(T_C - T_T\right) \tag{3}$$

$$\frac{\partial T_S}{\partial t} = \frac{h_S a_S}{A_S \rho_S C_{P_S}}\left(T_C - T_S\right) \tag{4}$$

## STATIC OPTIMIZATION OF HENs

With regard to a HEN, usually, the control objective is to keep target-stream temperatures either exactly at the nominal conditions for which the HEN was designed (hard targets) or as close as possible to the nominal conditions (soft targets). Generally, for an arbitrary disturbance, all of the hard targets may not be satisfied since flexibility (disturbance-rejection ability) at/around this nominal design point is not considered during traditional static design. Instead, an alternative operating point around the nominal operating conditions may be targeted. However, not all operating points around the nominal conditions are feasible. Therefore, for a specified/measured disturbance, feasible targets must be selected. Once this feasible operating point is identified, the remaining control task is to drive the system from its nominal operating conditions to the new feasible operating point (post-disturbance target) after a disturbance. The necessary degrees of freedom to achieve these control objectives can be obtained either from split streams which may be present in the original design of the HEN or by the addition of bypass streams which may require a retrofit design.

The static optimization (steady-state optimal control) problem can be defined as follows. When the HEN experiences disturbances in source-stream temperatures, $\Delta T^s_{H,n}$ and $\Delta T^s_{C,n}$, find a set of bypass fractions, $u_{H,n}$ and $u_{C,n}$, that will minimize the total deviations in the hot- and cold-stream target temperatures, $\sum\left\{\left|\Delta T^t_{H,n}\right| + \left|\Delta T^t_{C,n}\right|\right\}$, subject to hot and cold target-temperature control constraints (soft/range or hard), $\Delta T^t_{H,n}$ and $\Delta T^t_{C,n}$, such that the HEN remains feasible.

This static optimization problem can be formulated via infeasible-path (Konukman et al., 1995) or feasible-path (Boyacı, 1995) approaches; the latter approach that bases on an analytic solution, avoids the use of log-mean temperature differences, and thus, allows removal of some thermodynamic constraints.

## CENTRALIZED OPTIMAL CONTROL OF HENs

Referring to the optimization formulation described above, when the set of external disturbances entering the HEN are measured, optimal values of the target temperatures and bypass openings that guarantee steady-state feasibility of the new operating point can be calculated. This optimal solution also guarantees that the constraints on target-stream temperatures are satisfied as much as the network structure allows (more likely for soft, but less likely for hard constraints).

The remaining task is to determine how to apply in time the set of optimal bypass openings (controls) so that the HEN's dynamic response in reaching the final steady-state feasible operating point is acceptable. For this purpose, two different centralized optimal-control schemes, depicted in Fig. 1, are proposed.

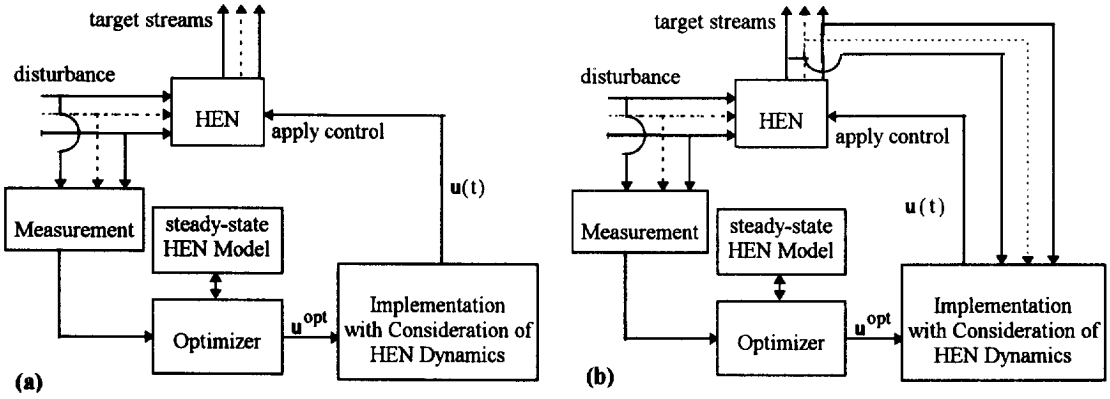



Fig. 1. Schematic of the centralized optimal open-loop (a) and closed-loop (b) control logic of HENs.

Fig. 1.a is a schematic of an optimal open-loop control logic of HENs, where there is no feedback of the state variables (target temperatures). According to this scheme, temperatures (disturbances) of the source streams entering the HEN are measured (sampled) at discrete time intervals. These measurements are fed to the optimizer that performs the static optimization mentioned above by referring to the steady-state algebraic model of the HEN. The optimizer finds the optimal values of the bypass fractions, $\mathbf{u}^{opt}$, such that the target-temperature constraints, that define the exact location (hard target) and/or allowable range (soft target) of the desired feasible steady-state operating point, are satisfied. Assuming that there is no mismatch between the static model of the optimizer and the process (HEN), the remaining task, at this point, is to find how to implement these optimal bypass opening as a function of time. Since, as mentioned earlier, there is no offset between the predictions of the algebraic and dynamic models at steady-state, the bypasses can be opened up to their optimal values either instantaneously or as a function of time; e.g., as a ramp starting from their nominal values, $\bar{u}$, which may be zero, towards their optimal values, $\mathbf{u}^{opt}$, within $\tau_n$ time interval, as represented by Eq.5.

$$u_n(t) = \begin{cases} \bar{u}_n + \left(u_n^{opt} - \bar{u}_n\right)\left(\dfrac{t}{\tau_n}\right) & \text{,if } t < \tau_n \\ u_n^{opt} & \text{, if } t \geq \tau_n \end{cases} \quad (5)$$

Fig. 1.b is a schematic of an optimal closed-loop control logic of HENs, where there is feedback of the state variables (target temperatures). Even though the implementation is similar to that of the open-loop scheme, the pairing problem among the bypasses and target temperatures must be solved before implementation of the closed-loop scheme. As one can imagine, there may arise many pairing possibilities even for a simple HEN. The function proposed for the implementation of the optimal controls in the centralized closed-loop optimal-control algorithm is given by Eq.6. Two different subscripts, *m* and *n*, are used to designate the bypassed exchanger number (*m*) and the exchanger number (*n*) the effluent-stream temperature, $T_n^e$, of which is used for feedback information. In such a centralized optimal-control scheme, all bypasses should be matched with at least one effluent stream of a target exchanger. Failure to connect even one manipulated variable to any of the pseudo-controlled variables, $T_n^e$, may drive the HEN away from the post-disturbance feasible operating point determined by the optimizer. According to implementation function, Eq.6, initially (nominal operating point at t=0), the feedback temperatures, $T_n^e(0)$, are equal to their values at the nominal operating point, $\bar{T}_n^e$, and, therefore, the bypass openings, $u_m(0)$, are equal to their nominal values, $\bar{u}_m$, (which may be zero). As the effect of the disturbance propagates through the HEN, the feedback temperatures, $T_n^e(t)$, and thus, the manipulated variables, $u_m(t)$, change. As the system moves towards the feasible operating point determined by the optimizer, the feedback temperatures approach to their optimal values, $T_n^{e,opt}$. At this post-disturbance steady-state operating point, the feedback temperatures attain the value of $T_n^{e,opt}(\infty)$, and thus, the optimal bypasses, $u_m^{opt}$, are reached. In this way, both the feedback temperatures and bypass openings simultaneously reach their optimal/feasible values.

$$u_m(t) = \bar{u}_m + \left(u_m^{opt} - \bar{u}_m\right)\left(\frac{T_n^e(t) - \bar{T}_n^e}{T_n^{e,opt}(\infty) - \bar{T}_n^e}\right) \quad (6)$$

**EXAMPLE**

As a simple example, consider the HEN given in Fig.2. The nominal values of the source and target temperatures, and the heat-capacity-flow-rates are indicated in the figure. The design was based on a constant overall heat-transfer coefficient of 1000 W/m$^2$-K for all the exchangers. Key design specifications can be found in Boyacı (1995). The bypass streams are shown with dashed lines, but the original design does not contain bypass streams; without any disturbance their values (nominal bypass fractions) are zero.

The control problem is to be able to keep the temperatures of the hot-target streams from the first and third exchangers and the cold-target stream of the third exchanger within ±1 K of their nominal values, and that of the cold-target stream from the second exchanger within ±2 K of its nominal value, when the HEN receives a −4 K disturbance in the temperature

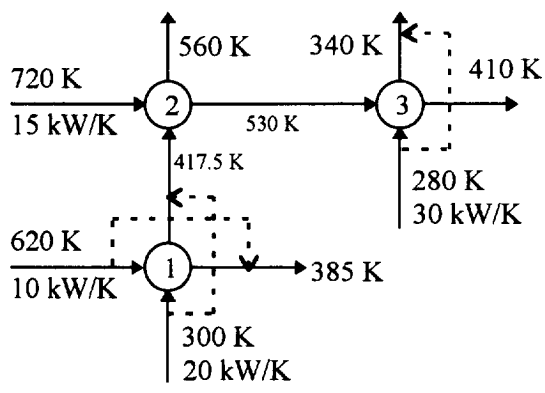



Fig. 2. A sample HEN

of the cold source stream to the first exchanger. The uncontrolled response of the HEN is shown in Fig. 3 as deviations of target temperatures from their nominal (zero-disturbance) values. The hot-target temperature from the first exchanger exceeds the desired control range of ±1 K.

Fig. 4 shows the response of the HEN under open-loop control when all bypasses are opened linearly in time from their nominal values, which are zero, to their optimal values calculated from the static optimization ($u^{opt}_{H,1}$=0.019919, $u^{opt}_{C,1}$=0.000468, $u^{opt}_{C,3}$=0.055962) using the package ADS (Vanderplaats, 1984), within $\tau_n$=10 s time interval, according to Eq.5. The hot-target temperature from the first and the cold-target temperature from the third exchangers violate their desired control-range constraints of ±1 K, temporarily. A similar but more severe temporal violation is also observed when all bypasses are opened instantaneously (as soon as the disturbance is measured; $\tau_n$=0 s). However, with time, these temporal violations disappear and the new feasible operating point, as predicted by the static optimizer, is reached. Fig. 5 shows the response of the HEN under open-loop control when all bypasses are opened within $\tau_n$=80 s time interval. The response of the HEN approaches to that of the uncontrolled HEN with such a slow rate of opening of the bypasses, and thus, the hot-target temperature from the first exchanger violates its control-range constraint. Again, with time, the temporal violations disappear and the new feasible operating point as predicted by the static optimizer is finally reached. Fig. 6 shows the response of the HEN under open-loop control when all bypasses are opened with a common optimal rate; within $\tau_n^{opt}$ =32 s, as determined from the solution of the dynamic optimization problem described by the objective function, Eq.7 (minimization of the integral of absolute deviations of target temperatures of the HEN from their nominal values), and the constraints, Eq.8 (all target temperatures of the HEN should be within their specified control ranges for all times). As can be seen, temporal violations are eliminated.

$$\underset{\tau_n}{minimize} \int_0^{t^\infty} \sum \left| T_n^t(t) - \bar{T}_n^t \right| dt \tag{7}$$

$$\left(\bar{T}_n^t - \Delta T_n^t\right) \le T_n^t(t) \le \left(\bar{T}_n^t + \Delta T_n^t\right) \tag{8}$$

Fig. 7 shows the response of the HEN under closed-loop control when the bypasses on the hot and cold sides of the first exchanger were both matched with the hot effluent (before bypass mixing) of the first exchanger and the bypass on the cold side of the third exchanger was matched with the target (after bypass mixing) of the third exchanger, in accordance with Eq.6. The pairing was arbitrary, and no temporal violations are observed for this particular case.

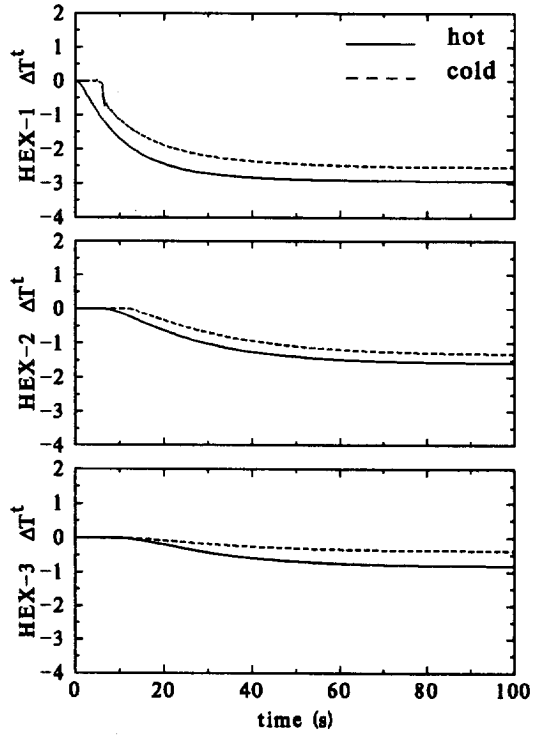



Fig. 3. Uncontrolled dynamic response of the HEN.

## CONCLUSIONS

The centralized open-loop control algorithm, with the use of ramp function for the opening of the bypasses, resulted in a very satisfactory dynamic response of the HEN and the temporal violations of the soft control-range constraints were prevented by the optimal tuning of the rate with which the bypasses were opened.

The centralized closed-loop control algorithm, where the bypasses were opened as dependent on the pseudo-controlled target-stream temperatures, resulted in a smoother dynamic response of the HEN. However, the closed-loop algorithm, in its presented form, may not always prevent temporal violation of the soft control-range constraints.

The open-loop and close-loop algorithms both do prevent violations of the hard targets at any feasible post-disturbance steady-state operating point.

Both of the proposed centralized steady-state-model-based optimal-control algorithms are promising in controlling HENs as long as the HENs have enough margin of steady-state flexibility.

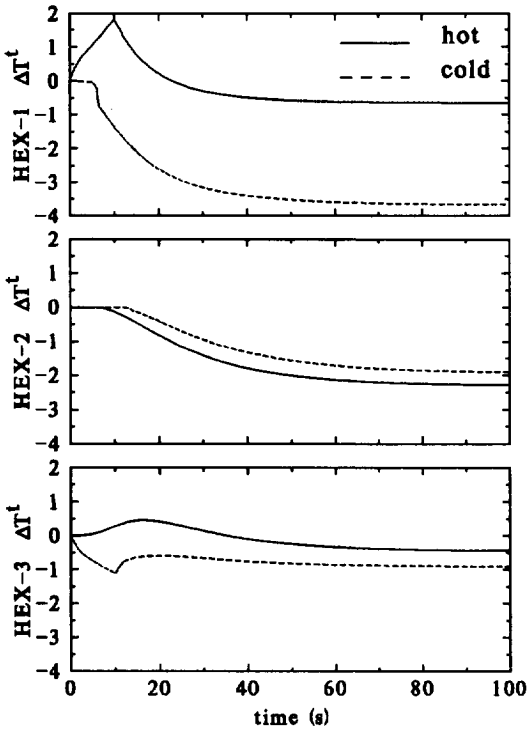



Fig. 4. Open-loop control of the HEN with $\tau_n$=10 s.

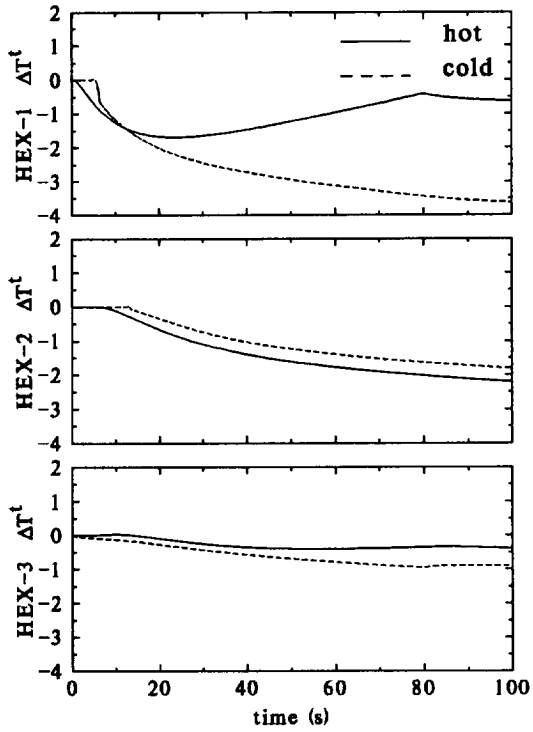



Fig. 5. Open-loop control of the HEN with $\tau_n$=80 s.

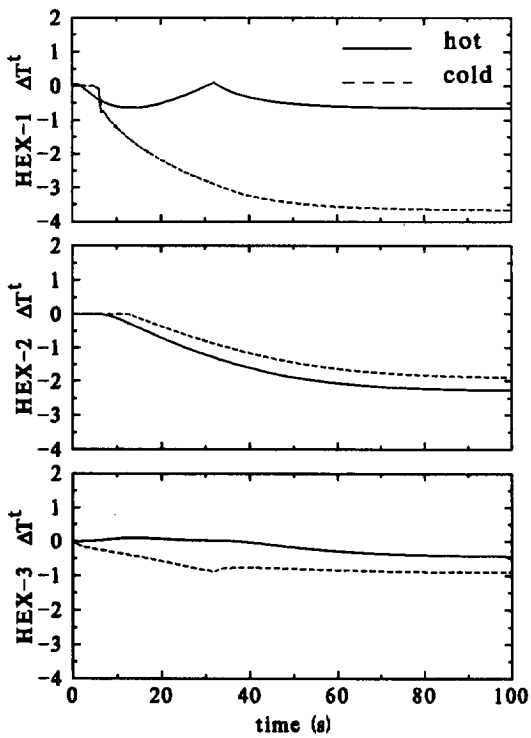



Fig. 6. Open-loop control of the HEN with $\tau_n^{opt}$ =32 s.

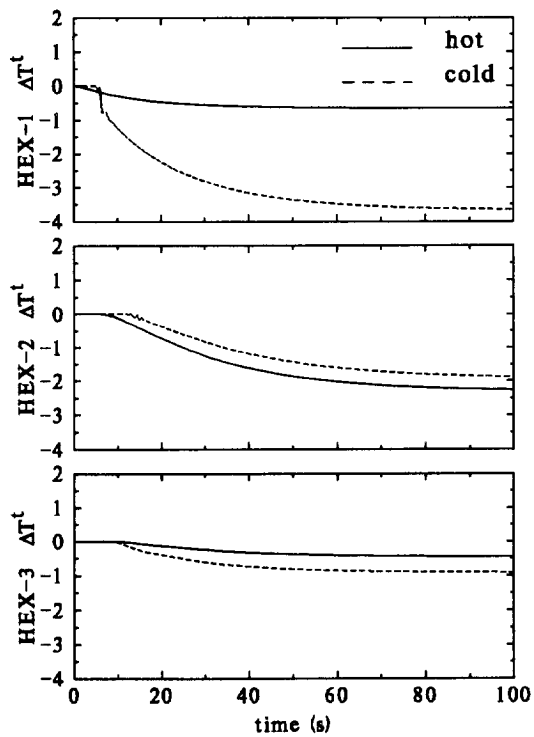



Fig. 7. Closed-loop control of the HEN.

## NOTATION

a = surface area of tube per unit length
A = cross-sectional area
$C_p$ = heat capacity
h = individual heat transfer coefficient
$N_t$ = number of tubes
t = time
T = temperature
$\Delta T$ = deviation in temperature
u = bypass fraction
**u** = vector of bypass fractions
v = velocity
z = axial position
$\infty$ = steady-state value

*Greek Letters*
$\rho$ = density
$\tau$ = rate of bypass opening

*Subscript*
C = cold (shell) side
H = hot (tube) side
m = $m^{th}$ bypass stream
n = $n^{th}$ heat exchanger
S = shell wall
T = tube wall

*Superscript*
e = effluent (before bypass mixing)
opt = optimum
s = source
t = target
$\bar{}$ = nominal value

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